460-2 International Economics

Lecture notes 4: Nominal rigidities

1 Overvaluation and multiple equilibria

- We build on the model developed in Notes 2
- Suppose the central bank is committed to a fixed exchange rate

$$\mathcal{E}_t = \bar{\mathcal{E}}$$

- We want to study how this committment can come under attack, if inflation expectations are out of line
- Consider a version of the model with two group of firms
- A mass α cannot change price, price is pre-set at $\bar{P}_h$
- A mass $1-\alpha$ (flex price firms) can change price at date 0
- Game at date 0:
  - Flex price firms set price $\hat{P}_{h0}$ forming expectations about C_0 and N_0
  - Central bank sets i_0 and $\mathcal{E}_0$ and quantities are determined
- When setting $\hat{P}_{h0}$ firms are also forming expectations about other firms' prices

1.1 Equilibrium

- Backward induction, given $\hat{P}_{h0}$ solve the central bank problem
- Price of home good is

$$P_{h0} = \left(\alpha \bar{P}_h^{1-\varepsilon} + (1-\alpha)\hat{P}_{h0}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

- Given total demand Y_0 for home goods the demand for the goods produced by fix and flex firms are

$$\left(\frac{\bar{P}_h}{P_{h0}}\right)^{-\varepsilon} Y_0 \text{ and } \left(\frac{\hat{P}_{h0}}{P_{h0}}\right)^{-\varepsilon} Y_0$$

- So aggregating and using linearity of the technology we have that total labor demand is

$$N_0 = J_0 Y_0$$

where

$$J_0 \equiv \alpha \left(\frac{\bar{P}_h}{P_{h0}} \right)^{-\varepsilon} + (1-\alpha) \left(\frac{\hat{P}_{h0}}{P_{h0}} \right)^{-\varepsilon}$$

- By choosing the nominal interest the central bank can choose any triple C_0, p_0 and Y_0 that satisfies

$$C_0 = p_0^{-\omega} \quad (1)$$
$$Y_0 = p_0^{-1} \quad (2)$$

exactly as in Notes 2

- Moreover the value of B_1 and the continuation welfare are independent of central bank policy so we can focus on welfare at date 0

$$U_0 = \log C_0 - \frac{\psi}{1+\phi} N_0^{1+\phi}$$

- Expressing it in terms of Y_0 we have

$$\omega \log Y_0 - \frac{\psi}{1+\phi} \left(J_0 Y_0 \right)^{1+\phi}$$

- If the central bank decides to float, its optimality condition is

$$\frac{\omega}{Y_0} = \psi J_0^{1+\phi} Y_0^{\phi}$$

- That is, the central bank best response is

$$Y_0 = (\omega/\psi)^{\frac{1}{1+\phi}} J_0^{-1}$$

- If central bank sticks to peg then

$$p_0 = \frac{P_{h0}}{\bar{\mathcal{E}}}$$

- Gain from floating

$$\Delta W(\hat{P}_h) = \max_Y \left\{ \omega \log Y - \frac{\psi}{1+\phi} \left(\mathcal{J}(\hat{P}_h) Y \right)^{1+\phi} \right\} - \left[\omega \log \mathcal{Y}(\hat{P}_h) - \frac{\psi}{1+\phi} \left(\mathcal{J}(\hat{P}_h) \mathcal{Y}(\hat{P}_h) \right)^{1+\phi} \right]$$

- Go backward to price setters optimality
- Price setters choose prices in anticipation of C_0,N_0,$\mathcal{E}_0$
- Optimality of price setters, together with equilibrium wages

$$\hat{P}_{h0} = P_0 C_0 N_0^{\phi}$$

where

$$P_0 = P_{h0}^{\omega} \mathcal{E}_0^{1-\omega}$$

- Assume

$$\omega = \psi$$

so if $\hat{P}_{h0} = \bar{P}_h = P_{h0}$ it is optimal for the central bank to implement the flexible price allocation

$$Y_0 = C_0 = p_0 = 1$$

- Assume

$$\bar{P}_h / \bar{\mathcal{E}} > 1$$

so currency is initially overvalued

1.2 Multiple equilibria

- Conjecture: equilibrium with

$$\hat{P}_{h0} = \bar{P}_h = P_{h0}$$

- Then $J_0 = 1$ and gain from floating is

$$\Delta W_{float} = \omega \log 1 - \frac{\psi}{1+\phi} - \left[\omega \log \frac{\bar{\mathcal{E}}}{\bar{P}_h} - \frac{\psi}{1+\phi} \left(\frac{\bar{\mathcal{E}}}{\bar{P}_h} \right)^{1+\phi} \right]$$

- Price setters optimality holds because they expect $C_0 = N_0 = 1$ and $\mathcal{E}_0 = P_{h0} = \bar{P}_h$

$$\hat{P}_{h0} = P_0 C_0 N_0^{\phi}$$

where

$$P_0 = P_{h0}^{\omega} \mathcal{E}_0^{1-\omega} = \bar{P}_h$$

- Suppose cost of floating is κ and satisfies

$$\kappa < \Delta W_{float}$$

then we have an equilibrium

- Can we have also an equilibrium with fixed exchange rates?

- Now price setters anticipate

$$C_0 = \left(\frac{\bar{\mathcal{E}}}{P_{h0}}\right)^{\omega}$$

$$Y_0 = \frac{\bar{\mathcal{E}}}{P_{h0}}$$

and

$$J_0 = \left[\alpha \bar{P}_{h0}^{-\varepsilon} + (1-\alpha)\,\hat{P}_{h0}^{-\varepsilon}\right] P_{h0}^{\varepsilon}$$

and

$$P_0 = P_{h0}^{\omega} \bar{\mathcal{E}}^{1-\omega}$$

- So we have

$$\begin{aligned}\hat{P}_{h0} \quad &= \quad P_0 C_0 N_0^{\phi} = P_{h0}^{\omega} \bar{\mathcal{E}}^{1-\omega} \left(\frac{\bar{\mathcal{E}}}{P_{h0}}\right)^{\omega} \left(\left[\alpha \bar{P}_{h0}^{-\varepsilon} + (1-\alpha)\,\hat{P}_{h0}^{-\varepsilon}\right] P_{h0}^{\varepsilon} \frac{\bar{\mathcal{E}}}{P_{h0}}\right)^{\phi} = \\ &= \bar{\mathcal{E}}^{1+\phi} \left(\frac{\alpha \bar{P}_{h}^{-\varepsilon} + (1-\alpha)\,\hat{P}_{h0}^{-\varepsilon}}{\alpha \bar{P}_{h}^{1-\varepsilon} + (1-\alpha)\,\hat{P}_{h0}^{1-\varepsilon}}\right)^{\phi}\end{aligned}$$

- Graphically we can see this has unique fixed point and

$$\hat{P}_{h0} < \bar{\mathcal{E}} < \bar{P}_h$$

which implies

$$\frac{P_{h0}}{\bar{\mathcal{E}}} < \frac{\bar{P}_{h0}}{\bar{\mathcal{E}}}$$

- So output if fixed expected and fixed is realized is higher than output if float is expected and fixed is realized
- If fixed is expected there is some internal devaluation that helps
- This suggests that ΔW_{fix} will be lower than ΔW_{float}
- There are added complications in proving this inequality, due to the presence of J
- But numerically I always got $\Delta W_{fix} < \Delta W_{float}$
- Moreover the distance between the two depends on the initial degree of overvaluation, if $\frac{\bar{P}_{h0}}{\bar{\mathcal{E}}} = 1$ then $\Delta W_{fix} = \Delta W_{float} = 0$
- So it's possible to find a κ such that

$$\Delta W_{fix} < \kappa < \Delta W_{float}$$

so we have two equilibria